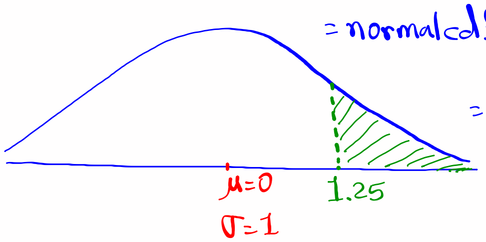
Statistics

Lecture 15

Feb 19-8:47 AM

Find $P(Z > 1.25)$

$= \text{normalcdf}(1.25, E99, 0, 1)$

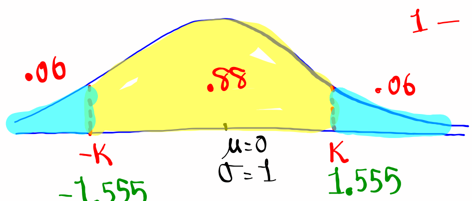


$= \boxed{.106}$

Find K such that $P(-K < Z < K) = .88$

$1 - .88 = .12$

$.12/2 = .06$



$K = \text{invNorm}(.94, 0, 1) = \boxed{1.555}$

Left Area

$.06 + .88 = .94$

Feb 8-4:31 PM

t - Dist.

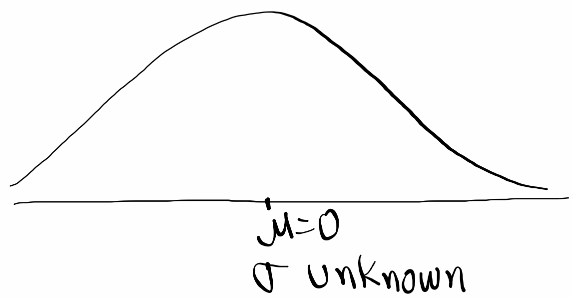
Data dist. is symmetric, bell-shape, with total area = 1

Mean $\mu = 0$, Standard deviation σ is unknown

It comes with degrees of freedom df

we use tcdf and invT

You can find them $\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$



Feb 8-4:37 PM

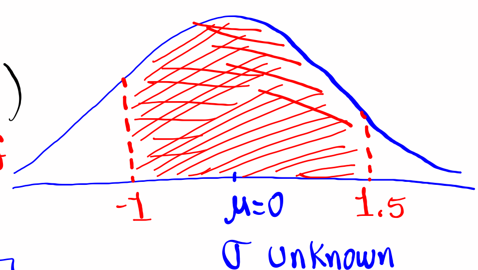
Find $P(-1 < t < 1.5)$ with df=15.

$\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$ ↓

$= \text{tcdf}(-1, 1.5, 15)$

Lower upper df

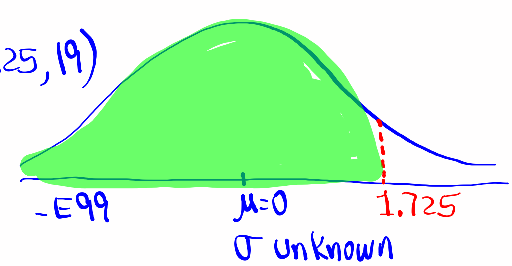
$= \boxed{.756}$



Find $P(t < 1.725)$ with df=19.

$= \text{tcdf}(-E99, 1.725, 19)$

$= \boxed{.950}$



Feb 8-4:40 PM

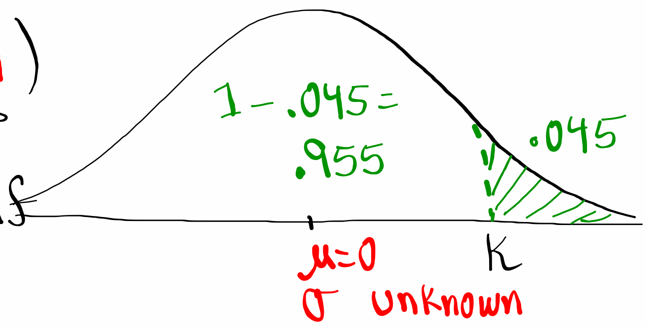
Find K such that $P(t > K) = .045$ with $df = 14$.

Right area

$K = \text{invT}(.955, 14)$

Left Area, df

$\approx \boxed{1.821}$

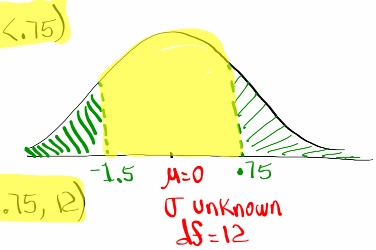


Feb 8-4:46 PM

Find $P(t < -1.5 \text{ OR } t > .75)$ with $df = 12$.

$= 1 - P(-1.5 < t < .75)$

Total Area



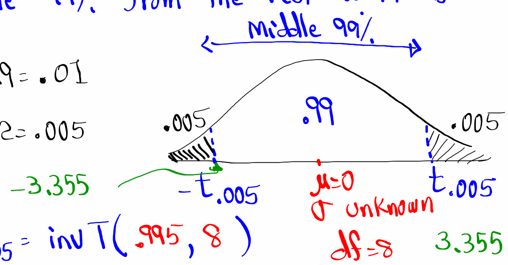
$= 1 - \text{tcdf}(-1.5, .75, 12)$

$= \boxed{.314}$

Find two t-values that separate the middle 99% from the rest with $df = 8$.

$1 - .99 = .01$

$.01 \div 2 = .005$



$t_{.005} = \text{invT}(.995, 8)$

$= \boxed{3.355}$

Feb 8-4:50 PM

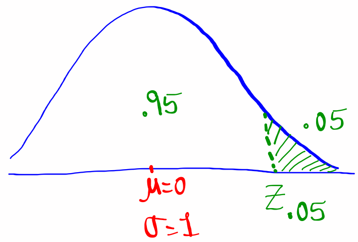
Find $Z_{.05}$

Right-Tail Area

$\frac{\alpha}{2} = .05$

$\boxed{\alpha = .1}$ Sig. level

$1 - \alpha = .9 \rightarrow 90\%$ C-level



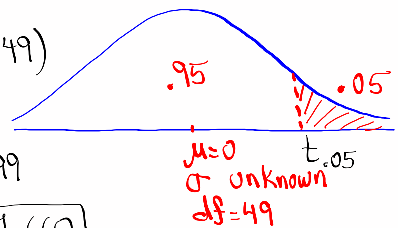
$Z_{.05} = \text{invNorm}(.95, 0, 1)$

$= \boxed{1.645}$

Find $t_{.05}$ with $df = 49$

$t_{.05} = \text{invT}(.95, 49)$

$= \boxed{1.677}$



Redo with $df = 99$

$\text{invT}(.95, 99) = \boxed{1.660}$

Redo with $df = 999 \Rightarrow \boxed{1.646}$

As df increase ⇒ $t \approx Z$

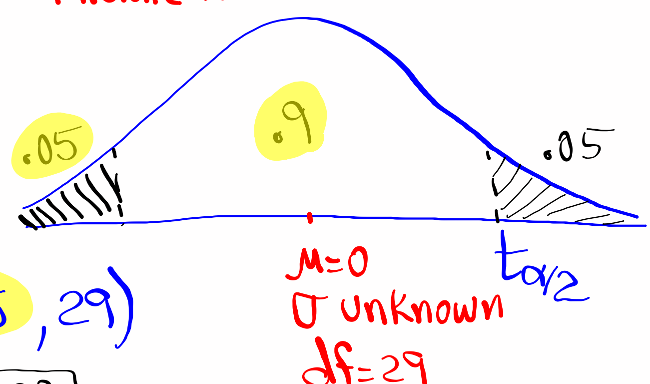
Feb 8-4:58 PM

Find $t_{\alpha/2}$ for C-level .9 with $df = 29$.

Middle Area

$1 - .9 = .1$

$.1 \div 2 = .05$



$t_{\alpha/2} = \text{invT}(.95, 29)$

$= \boxed{1.699}$

Feb 8-5:06 PM

What is degrees of freedom?

It varies by topic.

25 Donuts

24 people had choices, df = 24	Stephanie	has	25 choices.
	Izzy	"	24 "
	Adit	"	23 "
	⋮	⋮	⋮
	Michael (Last person)	has	0 choices (1 donut left)

I have 7 clean T-Shirts.

Monday	7 choices.	df = 6
Tuesday	6 "	
Wed.	5 "	
⋮	⋮	
Sunday	0 choices (1 clean T-Shirt)	

Feb 8-5:09 PM

Estimating Pop. mean with σ Unknown

$$\bar{x} - E < \mu < \bar{x} + E$$

Margin of error $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $\quad$ $t_{\alpha/2} \to df = n - 1$

15 students had a mean age of 30 Yrs with standard deviation of 8.

$n = 15$, $\bar{x} = 30$, $s = 8$

Find $\boxed{90\%}$ Conf. interval for the mean age of all students.

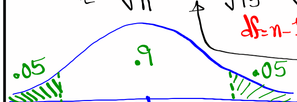
$$\bar{x} - E < \mu < \bar{x} + E$$

$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} = 1.761 \cdot \frac{8}{\sqrt{15}} \approx 3.6 \approx 4$ $\quad$ $df = n - 1$

$30 - E < \mu < 30 + E$

$30 - 4 < \mu < 30 + 4$

$\boxed{26 < \mu < 34}$



$\mu = 0$

σ unknown

$df = 14$

$t_{\alpha/2} = \text{invT}(.95, 14) = \boxed{1.761}$

Let's verify this by

TInterval $\quad$ $26 < \mu < 34$

[STAT] [TESTS] $\quad$ inpt: [Stats]

Feb 8-5:16 PM

10 randomly selected nurses had a mean monthly salary of \$6750 with standard deviation of \$450.

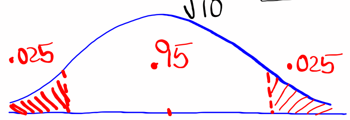
$n = 10$, $\bar{x} = 6750$, $S = 450$

Find $\boxed{\text{Conf. interval}}$ for the mean monthly salary for all nurses.

→ NO C-level
⇒ use .95

$E = t_{\alpha/2} \cdot \dfrac{S}{\sqrt{n}}$

$= 2.262 \cdot \dfrac{450}{\sqrt{10}} \approx \boxed{322}$



$\mu = 0$
σ unknown
$df = n-1 = 10-1 = 9$

$t_{\alpha/2} = \text{invT}(.975, 9)$
$= 2.262$

$\bar{x} - E < \mu < \bar{x} + E$

$6750 - 322 < \mu < 6750 + 322$

$\boxed{6428 < \mu < 7072}$

verify with TInterval

Feb 8-5:26 PM

I randomly selected 8 exams, here are the scores

72 88 90 65
100 75 95 60

1) Find $\bar{x}$ & S. Round to whole #.

$\bar{x} = 81$

$S = 15$

2) Find 99% conf. interval for the mean of all exams.

σ Known → ZInterval

$\boxed{\sigma \text{ unknown} \to \text{TInterval}}$

$df = n-1$
$= 8-1$
$= 7$

inpt: $\boxed{\text{Stats}}$

$\boxed{62 < \mu < 100}$

$E = \dfrac{100-62}{2} = \boxed{19}$

$\bar{x} = \dfrac{100+62}{2} = \boxed{81}$

Feb 8-5:36 PM

How to determine minimum Sample Size to do a Survey:

1) Proportion

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} \xrightarrow[\text{algebra work}]{\text{with some}} \boxed{n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2}$$

Always Round-up

If $\hat{p}$ & $\hat{q}$ unknown use .5 for each

$$\boxed{n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2}$$

2) Mean

$$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \xrightarrow[\text{Algebra work}]{\text{with some}} n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

if σ is unknown, use S

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E}\right)^2$$

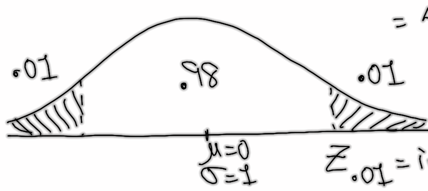
Feb 8-5:42 PM

Find minimum Sample Size needed if we wish to construct $\boxed{\text{98\% conf. interval}}$ for pop. proportion with point-estimate 25% and margin of error not to exceed 5%

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2 = (.25)(.75)\left(\frac{2.326}{.05}\right)^2$$

$= 405.7707$

$\boxed{n = 406}$



$Z_{.01} = \text{invNorm}(.99, 0, 1)$

Suppose $\hat{p}$ & $\hat{q}$ were unknown

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{2.326}{.05}\right)^2 = 541.0276$$

$\boxed{n = 542}$

Feb 8-6:03 PM

Find minimum sample size needed to construct 90% Conf. interval for pop. proportion with Point-estimate 40% and error not to exceed 4%

$\hat{p} = .4$

$\hat{q} = .6$

$E = .04$

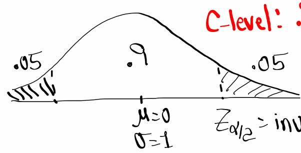
C-level: .9

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$$

$$= (.4)(.6)\left(\frac{1.645}{.04}\right)^2$$

$n = 405.90375$

$\boxed{n = 406}$



$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = 1.645$

Redo with $E = .08$

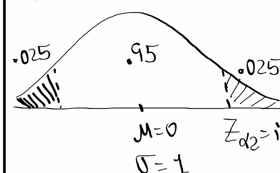
$$n = (.4)(.6)\left(\frac{1.645}{.08}\right)^2$$

$\boxed{n = 102}$

Now redo with 95% C-level

$$n = (.4)(.6)\left(\frac{1.960}{.08}\right)^2$$

$\boxed{n = 145}$



$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1)$

$= \boxed{1.960}$

Feb 8-6:12 PM

Find minimum sample size to construct 96% Conf. interval for pop. mean and margin of error not to exceed 10. Assume $\sigma = 25$.

$n = ?$

C-level: .96

$E = 10$

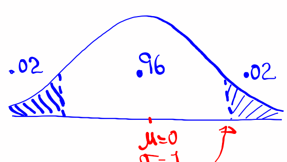
$\sigma = 25$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E}\right)^2$$

use S if σ unknown

$$= \left(\frac{2.054 \cdot 25}{10}\right)^2 = 26.368\ldots$$

$\boxed{n = 27}$

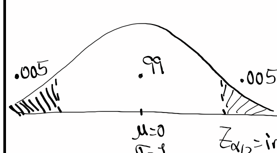


$Z_{\alpha/2} = \text{invNorm}(.98, 0, 1) = 2.054$

Redo with C-level: .99 and $E = 5$

$$n = \left(\frac{2.576 \cdot 25}{5}\right)^2 = 165.8944$$

$\boxed{n = 166}$



$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1)$

$= 2.576$

Feb 8-6:24 PM

Given $\boxed{\text{C-level}: .88}$, $E = 5$, $S = 10$

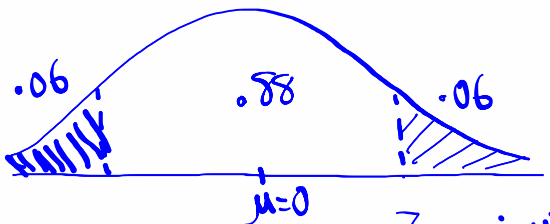
Find minimum Sample Size to construct conf. interval for Pop. mean.

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

If σ unknown → use S

$$n = \left(\frac{1.555 \cdot 10}{5} \right)^2 = 9.6721$$

$$\boxed{n = 10}$$



$\mu = 0$
$\sigma = 1$

$Z_{\alpha/2} = \text{invNorm}(.94, 0, 1)$

Feb 8-6:34 PM

Poisson Prob. dist.

It is on a fixed interval

Mean μ is given for that fixed interval.

$\sigma^2 = \mu$

$\sigma = \sqrt{\sigma^2}$

$P(x = \ ) = \text{Poissonpdf}$

$P(x \leq \ ) = \text{Poisson cdf}$

$P(x \geq \ ) = 1 - \text{Poissoncdf}$

Feb 8-6:39 PM

Alisa gets 36 Students in average per day for advising

$\mu = 36$

Fixed interval

$\sigma^2 = \mu = 36$

$\sigma = \sqrt{\sigma^2} = \sqrt{36} = 6$

Usual Range $\mu \pm 2\sigma = 36 \pm 2(6)$

$= 36 \pm 12 \Rightarrow \boxed{24 \text{ to } 48}$

P(She gets exactly 36 per Selected day)

$P(x = 36) = \text{PoissonPdf}(36, 36) \approx \boxed{.066}$

P(She gets less than 40 Students)

$P(x < 40) = P(x \leq 39) = \text{Poissoncdf}(36, 39)$

$= \boxed{.726}$

Feb 8-6:42 PM

Mike works as a dispatcher.

In average, he gets 16 Calls per hour.

$\mu = 16$

$\lambda = 16$

Fixed interval

$\sigma^2 = \mu = 16$

$\sigma = \sqrt{\sigma^2} = 4$

68% Range $\mu \pm \sigma = 16 \pm 4$

$\Rightarrow \boxed{12 \text{ to } 20}$

P(Mike gets at least 10 Calls per hour)

$P(x \geq 10) = 1 - P(x \leq 9)$

$= 1 - \text{Poissoncdf}(16, 9)$

9 10

$= \boxed{.957}$

P(Mike gets between 12 and 20 calls, inclusive, Per hour)

$P(12 \leq x \leq 20) = P(x \leq 20) - P(x \leq 11)$

$= \text{Poissoncdf}(16, 20) - \text{Poissoncdf}(16, 11)$

11 12 20

$= \boxed{.741}$

P(he gets 15 or 20 Calls Per hr)

$P(x = 15 \text{ OR } x = 20)$

$\text{Poissonpdf}(16, 15) + \text{Poissonpdf}(16, 20) = \boxed{.155}$

Feb 8-6:48 PM